Electromagnetic Model of a Radial-Resonator Waveguide Diode Mount

MAREK E. BIALKOWSKI, SENIOR MEMBER, IEEE

Abstract—**An electromagnetic model of a class of waveguide diode mounts that incorporate a radial resonator with the diode arbitrarily positioned is derived. The model is tested through comparison of numerical and experimental results for the input impedance of the mount. The tests show good agreement between theory and experiment and indicate that the model may be especially useful in investigating the effects of an off-centered positioning of the diode.**

I. Introduction

In THE SHF and EHF ranges the design of oscillators with Gunn or IMPATT diodes requires a special type of diode mount in order to overcome problems associated with conduction losses, which prevent oscillation. Use is made of a cavity created close to the diode. The cavity can have a shape different from that of the enveloping waveguide. For rectangular waveguides, practical reasons usually dictate the choice of a radial cavity of resonator. Two realizations of the radial resonator are (i) a cap resonator mount with a thin supporting post in a full-height waveguide [1], [2] and (ii) a thick post mount in a reduced-height waveguide [3], [4].

The design of oscillators with radial cavities requires some experimental or theoretical knowledge of the input impedance of the mount since it has to be matched to the impedance of the active device to obtain oscillation. Theoretical models of radial-resonator diode mounts which are adequate for most practical purposes have been presented in [4]–[7]. These models are restricted to a diode centrally located under a cap or thick post. This restriction is not severe since most radial-resonator mounts utilize the central location for the diode. However, these models are not suitable for studying the effects of diode misalignment. They also do not apply to those radial-resonator mounts in which the diode is intentionally off-centered and is often in a position at the circumference of the radial resonator [3]. The effects of off-centered positioning of the diode may be unimportant when the post or cap cross-sectional dimensions are small. This is not the case when these dimensions become comparable to the wavelength.

A simplified analysis of the effects of off-centered positioning of the diode in a radial cavity having dimensions comparable to the wavelength has been carried out in [11]. By using a simple model of an open-circuited radial line it was shown that off-centered positioning of the diode can produce resonances of radial azimuthal modes. It was also indicated that these resonances can affect in different ways the design of an oscillator or amplifier. Accurate analysis of the effects of off-centered positioning of the diode was not possible with this model since radiation from the radial resonator was neglected.

In the paper presented here an electromagnetic model of a radial-resonator diode mount is developed which takes into account radiation from the radial resonator so that a more accurate analysis of the effects of different diode positioning can be carried out.

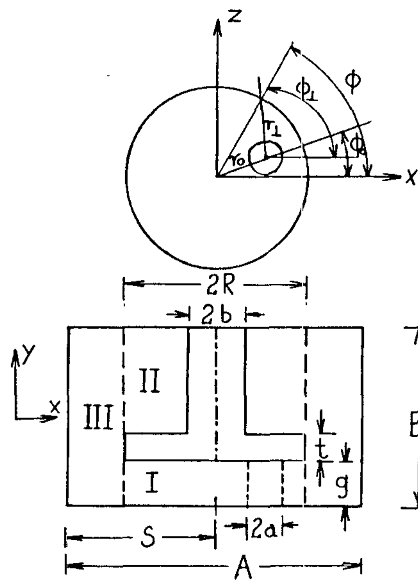

Fig. 1. Radial-resonator diode mount in a rectangular waveguide.

II. Method of Modeling

The configuration of the mount investigated is shown in Fig. 1. The mount and the waveguide are assumed to be formed by a perfect conductor. The structure can be viewed as composed of three cylindrical regions: I—below the cap where the diode is usually located, II—above the cap where the supporting post is located, and III—a waveguide region outside the cylindrical volume occupied

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The author is with the Department of Electrical and Electronic Engineering, James Cook University of North Queensland, Townsville, Qld. 4811, Australia.

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by the mount to which power from the diode is coupled. Each of these regions can be assumed to be filled with lossless dielectric material described by the real constants ϵ_1, ϵ_2, and ϵ_3.

The aim of the analysis is the determination of the driving-point impedance seen by the diode at an arbitrary position under the cap. This impedance is defined as the ratio of the voltage to the current at the cylindrical surface enveloping the diode [5]. In this definition the voltage and the associated electric field are assumed to be independent variables and the current is a variable to be determined.

To find the required current a three-dimensional electromagnetic problem has to be solved. For purposes of solving this field problem two cylindrical systems of coordinates are introduced. One (r, ϕ, y) is associated with the mount; the other (r_1, ϕ_1, y) is associated with the point at which the input impedance is calculated. The two systems are shown in Fig. 1.

In order to determine the driving-point impedance a field matching technique is used. The technique consists of three steps. In the first step field expansions with unknown coefficients in the individual regions are formed. In the second step the expansion coefficients and fields are determined by applying continuity conditions for the fields at the common boundary of the three regions. In the last step the input impedance is determined by using the usual voltage–current ratio definition. As the driving-point impedance has to be found at an arbitrary position, forming general expansions for the fields requires solving two scattering problems, one being internal and the other external to the cylindrical region $r < R$.

The internal problem for the radial regions below and above the cap can be considerably simplified if the heights of these regions can be assumed to be electrically small (smaller than half a wavelength). In this case the field below and above the cap can be approximated by a field which is uniform with height such that TM radial modes are sufficient to represent the field in both regions. The assumption is justified for post structures where the height of region II is zero or for a thick cap structure for which the heights of regions I and II are small. Further analysis is limited to these cases.

The rectangular waveguide region III can be regarded as a loaded radial guide. Since the field at the boundary $r = R$ of region III varies with angle ϕ and height y the TM modes are insufficient to satisfy the boundary conditions and both TM and TE radial modes must be used [10]. The only simplification used in deriving fields in region III results from the condition that the ϕ component of the electric field be zero at $r = R$, which is due to the assumption of fields uniform with respect to height in regions I and II and due to the condition that the tangential component of the electric field vanish on the conducting surface of the mount structure.

The problem of determining general forms of fields in regions I, II, and III can be considered as a scattering problem. The solutions to the internal and external scattering problems are presented below.

A. Fields Below the Cap

The field in the radial resonator below the cap can be considered as a superposition of two fields: one due to the voltage source applied at the cylindrical surface $r_1 = a$ (enveloping the diode) and the other due to induced sources located outside the cylinder $r = R$. Assuming that the height of the resonator is smaller than half a wavelength (which is usually the case), the field can be approximated by the uniform-with-height field, which can be expanded in terms of only TM radial waves [10]. The expansion coefficients can be found by considering the y component of the electric field.

The y component of the electric field can be sought in the form

$$E_y^{\mathrm{I}} = E_y^v + E_y^i \tag{1}$$

where E_y^v is the voltage source component and E_y^i is the induced source component. The voltage source component is associated with the wave traveling from the cylinder $r_1 = a$ outwards and is given by

$$E_y^v = \frac{V}{g}\frac{H_0^{(2)}(k_1 r_1)}{H_0^{(2)}(k_1 a)} \tag{2}$$

where $H_0^{(2)}$ is the Hankel function and k_1 is a wavenumber for the medium in volume 1.

The induced component E_y^i can be considered as the superposition of the subcomponents E_{yl}^i having dependence $e^{jl\phi}$ at $r = R$ and equal to zero at $r_1 = a$ and can be represented by

$$E_y^i = \sum_{l=-L}^{L} A_l \cdot E_{yl}^i. \tag{3}$$

The individual subcomponents E_{yl}^i can be represented in the form

$$E_{yl}^i = C_l \cdot H_0^{(2)}(k_1 r_1) + \sum_{p=-L}^{L} C_{lp} \cdot J_p(k_1 r) \cdot e^{jp\phi} \tag{4}$$

where J_p is the Bessel function and C_l and C_{lp} are expansion coefficients yet to be determined. The second term in (4) represents waves traveling from the cylinder $r = R$ inwards. The first term represents the wave which is reflected from the cylinder $r_1 = a$ and travels outwards. The wave reflected from the cylinder $r_1 = a$ is approximated by a radially symmetric wave. This approximation is based on the assumption that the radius a of the post is small. Note, that in expression (4) two cylindrical systems of coordinates are used to describe the field. Also note that in (4) and in the following expressions infinite series are truncated and include only $2L+1$ ϕ-harmonics.

The coefficients C_l and C_{lp} can be calculated by using conditions that E_y^i be zero at $r_1 = a$ and equal $e^{jl\phi}$ at $r = R$. The conditions can be written as follows:

$$\int_0^{2\pi} E_{yl}^i(r_1 = a)\, d\phi_1 = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi} E_{yl}^i(r = R) e^{-jp\phi}\, d\phi = \delta_{lp}$$

$$p = -L, \cdots, L \tag{5}$$

where δ_{lp} is the Kronecker delta. Note that only a condition on the average value of E^i_{yl} at $r_1=a$ is used. The higher order effects of the variation of E^i_{yl} at $r_1=a$ are neglected. Equations (5) lead to a system of linear algebraic equations which produce the following expressions for coefficients C_l and C_{lp}:

$$C_l = J_l(k_1 r_0) e^{jl\phi_0} \bigg/ \left\{ J_l(k_1 R) \sum_{p=-L}^{L} \frac{J_p^2(k_1 r_0) H_p^{(2)}(k_1 R)}{J_p(k_1 R)} - \frac{H_0^{(2)}(k_1 a)}{J_0(k_1 a)} \right\}$$

$$C_{lp} = \left(\delta_{lp} - C_l J_p(k_1 r_0) H_p^{(2)}(k_1 R) e^{-jp\phi_0}\right) / J_p(k_1 R). \tag{6}$$

Having derived the general form of the y component of the electric field in the resonator, the remaining components can be determined by using relationships which hold between the individual components of the TM radial field [10].

In determining the input impedance of the mount there is a need to know only the ϕ component of the magnetic field. For the TM field uniform in the y direction the ϕ component of the magnetic field is given by

$$H^{\mathrm{I}}_\phi = \frac{-j}{Z_1 k_1} \frac{\partial E^{\mathrm{I}}_y}{\partial r} \tag{7}$$

where Z_1 is the wave impedance for the medium in the resonator and $j=\sqrt{-1}$. By using (7) the ϕ component of the magnetic field at $r=R$ can be shown to be given by

$$H^{\mathrm{I}}_\phi(r=R) = \frac{-j}{Z_1} \sum_{l=-L}^{L} \frac{V}{g} \frac{H_l'^{(2)}(k_1 R)}{H_0^{(2)}(k_1 a)} J_l(k_1 r_0) e^{jl(\phi-\phi_0)} + \sum_{l=-L}^{L} A_l \sum_{m=-L}^{L} t_{lm} e^{jm\phi} \tag{8}$$

where

$$t_{lm} = C_l J_m(k_1 r_0) e^{-jm\phi_0} H_m'^{(2)}(k_1 R) + C_{lm} J_m'(k_1 R)$$

and the prime indicates a derivative. Similarly, the average value of the ϕ component of the magnetic field at $r_1=a$ can be shown to be given by

$$\frac{1}{2\pi} \int_0^{2\pi} H^{\mathrm{I}}_\phi(r_1=a)\, d\phi_1 = \frac{-j}{Z_1} \left\{ \frac{V}{g} \frac{H_0'^{(2)}(k_1 a)}{H_0^{(2)}(k_1 a)} + \sum_{l=-L}^{L} A_l \left(C_l H_0'^{(2)}(k_1 a) + J_0'(k_1 a) \cdot \sum_{m=-L}^{L} C_{lm} J_m(k_1 r_0) e^{jm\phi_0} \right) \right\}. \tag{9}$$

B. Fields Above the Cap

Volume II above the cap again forms a radial resonator structure. By making the assumption that the field does not vary along the height of the resonator, the field can again be represented by the TM radial waves [10]. For the centrally located post the scattering problem can be easily solved. The y component of the electric field is given by

$$E^{\mathrm{II}}_y = \sum_{l=-L}^{L} D_l e^{jl\phi} \cdot \frac{H_l^{(2)}(k_2 r) J_l(k_2 b) - J_l(k_2 r) H_l^{(2)}(k_2 b)}{H_l'^{(2)}(k_2 R) J_l(k_2 b) - J_l'(k_2 R) H_l^{(2)}(k_2 b)} \tag{10}$$

where k_2 is a wavenumber for region II. The ϕ component of the magnetic field is given by

$$H^{\mathrm{II}}_\phi = \frac{-j}{Z_2} \sum_{l=-L}^{L} D_l e^{jl\phi} \cdot \frac{H_l'^{(2)}(k_2 r) J_l(k_2 b) - J_l'(k_2 r) H_l^{(2)}(k_2 b)}{H_l'^{(2)}(k_2 R) J_l(k_2 b) - J_l'(k_2 R) H_l^{(2)}(k_2 b)}. \tag{11}$$

C. Fields Outside the Cylinder

It can be seen from the derivations of the field in volumes I and II that the only nonvanishing tangential components of the field at the common boundary $r=R$ with region III are the y component of the electric field and the ϕ component of the magnetic field. These components are continuous across the surface $r=R$. Region III can be regarded as a loaded parallel-plate radial guide with the field generally given in terms of TM and TE radial waves. Because the field in volume III varies with height and on the circumference of the cylinder $r=R$, the boundary conditions cannot be satisfied by the TM radial waves alone and both TM and TE waves must be used. (The representation of the field in terms of TM radial waves is possible only when the field is axially symmetric [5].)

In order to determine the input impedance of the mount, only expressions for the tangential components of the field at $r=R$ need be known. A convenient representation for the tangential components of the field at $r=R$ which uses scattering coefficients of the TM and TE waves is given by

$$E^{\mathrm{III}}_y = \sum_{n=0}^{N} \frac{\epsilon_{0n}}{B} \cos(k_{yn} y) \sum_{l=-L}^{L} FE_{nl} e^{jl\phi}$$

$$H^{\mathrm{III}}_y = \sum_{n=1}^{N} \frac{\epsilon_{0n}}{B} \sin(k_{yn} y) \sum_{l=-L}^{L} FH_{nl} \sum_{m=-L}^{L} f_{nlm} e^{jm\phi}$$

$$E^{\mathrm{III}}_\phi = \sum_{n=1}^{N} \frac{\epsilon_{0n}}{B} \sin(k_{yn} y) \cdot \sum_{l=-L}^{L} \left[FE_{nl} \cdot \frac{-k_{yn}(jl)}{R\Gamma_n^2} + FH_{nl} \right] e^{jl\phi}$$

$$H^{\mathrm{III}}_\phi = \sum_{n=0}^{N} \frac{\epsilon_{0n}}{B} \cos(k_{yn} y) \sum_{l=-L}^{L} FE_{nl} \sum_{m=-L}^{L} s_{nlm} e^{jm\phi} + FH_{nl} \sum_{m=-L}^{L} \frac{k_{yn}(jm)}{R\Gamma_n^2} f_{nlm} e^{jm\phi} \tag{12}$$

where ϵ_{0n} is the Neumann factor, $k_{yn}=n\pi/B$, and $\Gamma_n^2=k_3^2-k_{yn}^2$. FE_{nl}, FH_{nl} are expansion coefficients and s_{nlm} and f_{nlm} are scattering coefficients for TM and TE radial waves respectively.

The scattering coefficients are as yet unknown and have to be determined before setting continuity equations for the expansion coefficients FE_{nl} and FH_{nl}. The solution to this problem is not straightforward due to circular–rectangular boundaries of volume III. The difficulty associated with the circular–rectangular geometry of volume III can be overcome by using a filamentary approximation of the sources of TM and TE radial waves. The solution based on this type of approximation is presented in the Appendix.

D. Field Matching

At this stage all the required expansions for the field in volumes I, II, and at the boundary of volume III have been formed and the remaining part of the field-matching technique is the determination of the expansion coefficients. These coefficients can be found after applying the continuity conditions for the tangential components of the field at the common boundary $r=R$.

The continuity conditions imply the following system of functional equations:

$$E_y^{\mathrm{III}}=\begin{cases}E_y^{\mathrm{I}} & \text{for} \quad 0\leqslant y\leqslant g\\ 0 & \qquad\quad g\leqslant y\leqslant g+t\\ E_y^{\mathrm{II}} & \qquad\quad g+t\leqslant y\leqslant B\end{cases}$$

$$\begin{aligned}H_\phi^{\mathrm{I}}&=H_\phi^{\mathrm{III}} &&\text{for}\quad 0\leqslant y\leqslant g\\ H_\phi^{\mathrm{II}}&=H_\phi^{\mathrm{III}} &&\text{for}\quad g+t\leqslant y\leqslant B\\ E_\phi^{\mathrm{III}}&=0 &&\text{for}\quad 0\leqslant y\leqslant B.\end{aligned} \tag{13}$$

These equations can be reduced to a system of linear algebraic equations for the coefficients A_l, D_l, FE_{nl}, FH_{nl} after applying the standard Galerkin procedure [9]. In this procedure both sides of the equations in (13) are multiplied by the y or ϕ spatial harmonics and integrated within the bounds in which the orthogonality properties of the harmonics can be used. In order to reduce the number of unknowns, the coefficients FE_{nl}, FH_{nl} can be eliminated. First it can be easily noticed that coefficients FH_{nl} can be expressed in terms of coefficients FE_{nl} by using the condition that the ϕ component of the electric field be zero at $r=R$.

Finally coefficients FE_{nl} can be expressed in terms of coefficients A_l and D_l by using boundary conditions for the y component of the electric field and the ϕ component of the magnetic field at $r=R$. The resulting equations for coefficients A_l, D_l can be solved by using the standard Gauss elimination method.

E. Input Impedance of the Mount

Having determined the expansion coefficients of the fields, the driving-point impedance of the mount can be found by using the following expression:

$$Z_{\mathrm{IN}}=\frac{V}{I}=\frac{V}{a\int_0^{2\pi}H_\phi^{\mathrm{I}}(r_1=a)\,d\phi_1} \tag{14}$$

where $\int_0^{2\pi}H_\phi^{\mathrm{I}}(r_1=a)\mathrm{d}\phi_1$ is given by (9).

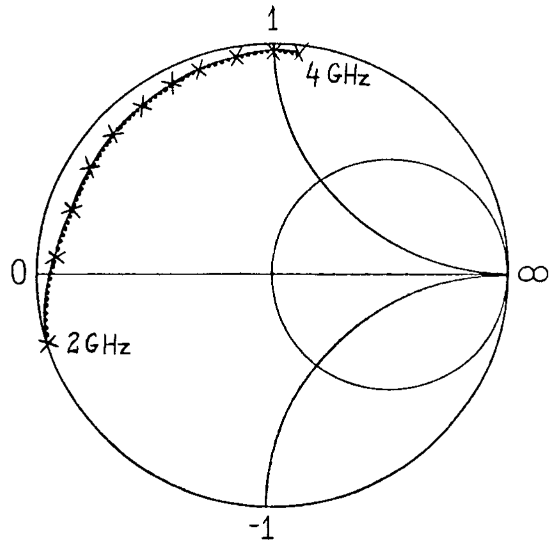

Fig. 2. Theoretical results for the input impedance of the centrally excited thick post mount being centrally positioned in the match-terminated S-band guide. Dimensions: $A=72$ mm, $B=34$ mm, $S=36$ mm, $R=b=17.5$ mm, $g=5$ mm, $t=29$ mm, $r_0=0$ mm. Results obtained by including: one ϕ harmonic $\cdots$; five ϕ harmonics ×——×. × indicate 0.2 GHz frequency step.

III. Experimental Verification

Based on the theory described above a computer algorithm for determining the input impedance of the radial-resonator waveguide mount with an arbitrary position of the diode has been developed. A number of experiments have been performed to investigate the limits of the model.

First, the model has been tested for mounts centrally excited. A study has been undertaken to investigate convergence of the solution for the input impedance versus the number of ϕ circumferential harmonics used in calculations. During the tests one, three, five, seven, and nine ϕ harmonics ($\exp(jn\phi)$ $n=0,1,2,\cdots$) were used for calculating the input impedance. The tests showed that the convergence of the solution was quite fast and that for five harmonics the error was less than 1 percent. Inclusion of seven and nine harmonics required considerably greater time for calculations with only negligible improvement in the accuracy for the impedance. Therefore to obtain accurate results it was decided to use only five ϕ harmonics in further calculations.

The results obtained by including one and five ϕ harmonics for the centered and off-centered cases of a thick post are presented in Figs. 2 and 3. Note that the values for the input impedance are normalized to 50 Ω. It can be seen that for the centrally positioned post the results for the input impedance are almost identical. The negligible difference cannot be observed without magnifying Fig. 2.

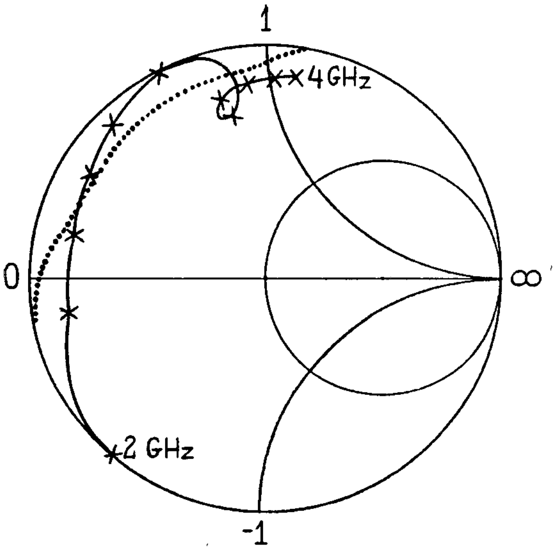

Fig. 3. Theoretical results for the input impedance of the centrally excited thick post mount being positioned close to the wall in the match-terminated S-band guide. Dimensions as in Fig. 2 but with $S = 18$ mm. Results obtained by including: one ϕ harmonic $\cdots$; five ϕ harmonics ×———×. × indicate 0.2 GHz frequency step.

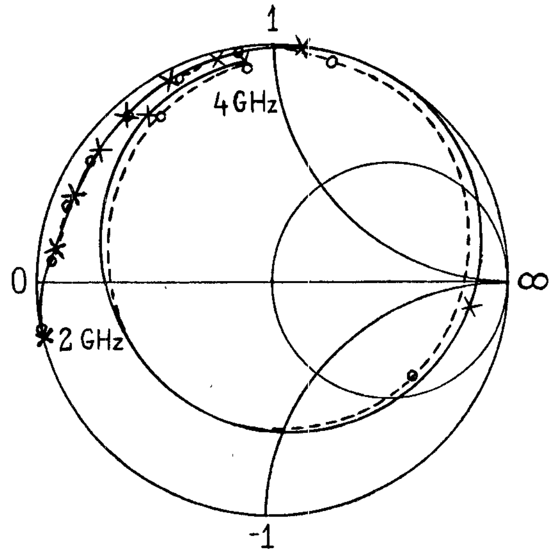

Fig. 5. Comparison between theoretical and experimental values of the input impedance of the off-center excited thick post mount in the match-terminated S-band guide. Dimensions as in Fig. 2 but with $S = 31$ mm, $r_0 = 5$ mm, $\phi_0 = 0°$. Theoretical results ×———×; experimental results —∘—∘∘. ∘× indicate 0.2 GHz frequency step.

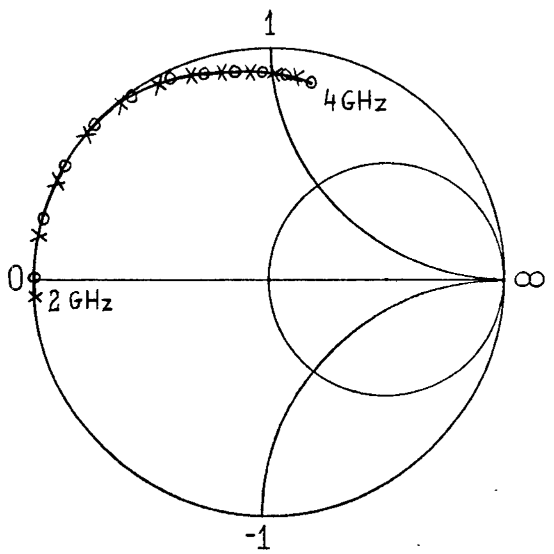

Fig. 4. Theoretical results for the input impedance of the centrally excited cap-resonator mount being centrally positioned in the match-terminated S-band guide. Dimensions: $A = 72$ mm, $B = 34$ mm, $S = 36$ mm, $R = 17.5$ mm, $b = 3.5$ mm, $g = 5$ mm, $t = 5$ mm, $r_0 = 0$ mm. Results obtained by including: one y harmonic in region II ×———×; four y harmonics —∘—∘∘. ∘× indicate 0.2 GHz frequency step.

For the post positioned off-center and located close to one of the waveguide walls the results depend on the number of circumferentially varying harmonics included in the calculations. In particular the results with five ϕ harmonics reveal the presence of the damped resonance at around 3.3 GHz. This resonance is not observed when the field in the cylindrical volume $r < R$ is approximated by an axial field.

Next, the assumption made in the present theory that the field above the cap can be approximated by the uniform-with-height field was tested. Tests were restricted to the case of a cap mount being centrally positioned in the waveguide and centrally excited. For this case the field in the vicinity of the mount could be assumed axially symmetric so that the algorithm developed in [5], which takes into account variation of the field in regions I and II, could be used. Comparison between results for the input impedance of the cap-resonator mount with a relatively high region above the cap, obtained with one y harmonic in region II using the algorithm developed here and four y harmonics ($\cos(k_{yn}y)$ $n = 0,1,2,3$) using the algorithm developed in [5], is shown in Fig. 4.

It can be seen that the results do not vary appreciably with the number of y harmonics used in the whole band of the single mode operation of the rectangular waveguide. The slight difference observed is apparent as shifting of the impedance points in the frequency domain.

In the following step the model was tested for mounts that were excited off center. These tests were supported by measurements of the input impedance of the mount. In order to avoid problems with mechanical tolerances, a large-scale model of the mount comprising a section of S-band waveguide (cross section: 72 mm×34 mm) with replaceable radial resonators was built.

To measure the input impedance of the mount a coaxial entry of 10 mm outside diameter with a 3 mm center conductor connected to an N-type plug was constructed at the center of the bottom wall of the waveguide. Thick posts and cap resonators could be attached to the demountable top wall of the waveguide. A generator with a slow frequency sweep was used so that no sharp resonances were overlooked. The impedances were recorded using an HP 8410 network analyzer.

The theoretical and experimental results for the thick post and cap-resonator mount, with dimensions the same as before but now with the off-center form of excitation, are presented in Figs. 5 and 6. Good agreement between

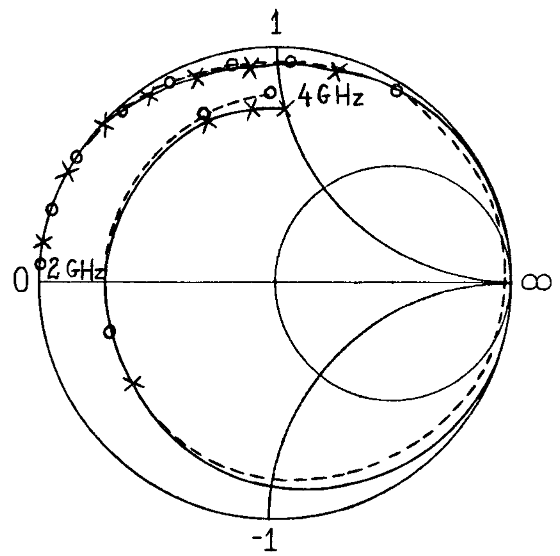

Fig. 6. Comparison between theoretical and experimental values of the input impedance of the off-center excited cap-resonator mount in the match-terminated S-band guide. Dimensions as in Fig. 4 but with $S = 31$ mm, $r_0 = 5$ mm, $\phi_0 = 0°$. Theoretical results ×——×; experimental results —∘—∘∘. ∘× indicate 0.2 GHz frequency step.

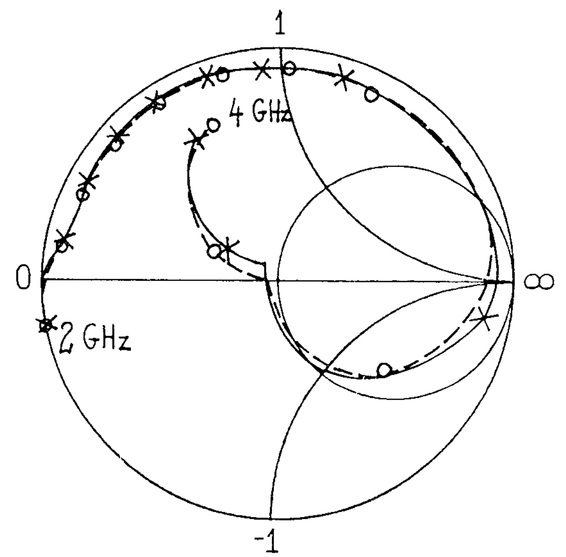

Fig. 7. Comparison between theoretical and experimental values of the input impedance for the off-center excited thick post Dimensions as in Fig. 2 but with $S = 31$ mm, $r_0 = 5$ mm, $\phi_0 = 45°$ Theoretical results ×—×–; experimental results –∘—∘. ∘× indicate 0.2 GHz frequency step.

theory and experiment can be noted. Although the excitations of the mounts are only slightly different from those considered before, the curves shown in Figs. 5 and 6 are significantly different from those in Figs. 2 and 4. These differences can be explained by resonances of the circumferentially varying TM radial modes, observed earlier in Fig. 3, which were fully developed for the off-centered form of excitation in the radial resonator. These resonances can easily be identified by using the theory of an open-circuited radial resonator [12]. According to the theory developed in [12], the only likely resonance to be produced in the 2–4 GHz band is due to the TM_{110} radial mode. Resonances of the next higher order modes (TM_{210} and TM_{010}) must be excluded since their resonant frequencies are above 6 GHz.

In order to gain further insight into resonances of the TM_{110} mode, another experiment, where the mount was off-centered with respect to the waveguide and the excitation was off-centered with respect to the radial cavity, was carried out. A comparison between experimental and theoretical results for the input impedance of the mount is shown in Fig. 7. It can be seen that in this case that resonances of two TM_{110} modes (indicated by two distinctive small arcs close to the center of the Smith chart) take place. One resonance is supported by the off-centered location of the mount versus waveguide walls and the second is due to the off-centered excitation of the radial resonator.

IV. Conclusions

An electromagnetic model of a radial-resonator waveguide diode mount with an arbitrary position of the diode has been presented. The model has been tested through comparison of the results for the input impedance obtained by means of the new model with those obtained from the earlier developed model or from measurements.

It has been found that for a centrally excitated radial resonator centrally positioned in a waveguide mount the new model is not advantageous relative to the simplified model of [5]. The results for the input impedance obtained by means of the two models are virtually the same.

The advantage of the new model is significant in situations in which the axial symmetry of the field in the region occupied by the mount is evidently disturbed. This is the case for the centrally excited mounts positioned close to the waveguide walls or for arbitrarily positioned mounts but with off-center excitation. A study with the off-center excited mounts has shown that the radial-resonator mount supports resonances of the circumferentially varying fields. Close to such resonant points the input impedance of the mount is very sensitive to variation in the position of the excitation, and even small offsets lead to quite large changes in values of the impedance.

The measurements performed on the physical models have shown that the new theoretical model quite accurately determines the input impedance in any case of excitation.

Appendix

In an electromagnetic field problem of a rectangular waveguide being excited by cylindrical sources, it is convenient to consider a rectangular waveguide as a loaded parallel-plate radial guide. In such a guide an arbitrary field can be represented by the superposition of radial waves that are transverse magnetic (TM) or transverse electric (TE) to the y axis of the guide.

In the problem of the radial-resonator mount the region $r > R$ is source free, and in accordance with the Schelkunoff field equivalence principle [10] the field in this region can be thought of as due to equivalent currents flowing on the cylindrical surface $r = R$. Because of the presence of the waveguide walls the waves produced by the currents are scattered. The solution to the scattering of both type of radial waves is presented below.

A. Scattering of TM Radial Waves

The TM radial waves existing outside the cylinder $r = R$ can be considered as due to radiation of a surface electric current of density J flowing on the cylinder $r = R$ in the y direction. Expression (12) shows that the scattering coefficient is defined as the magnitude of the nmth harmonic of the external (to the region $r < R$) ϕ component of the magnetic field when the y component of the electric field at $r = R$ is given by the nlth harmonic of unit magnitude.

Since the external ϕ component of the magnetic field is related to its internal component and the current density J by

$$H_\phi^+(r = R) = H_\phi^-(r = R) + J \tag{A1}$$

the determination of the scattering coefficients is related to the determination of the distribution of the current density J. This statement can further be developed as follows. The current density J corresponding to $E_y = E_{ynl}$, given by the nlth harmonic of unit magnitude, can be expanded into the series

$$J_{nl} = \sum_{m=-L}^{L} J_{nlm} e^{jm\phi} \cos(k_{yn} y) \tag{A2}$$

where J_{nlm} are expansion coefficients.

For the assumed value of the y component of the electric field at $r = R$ the y component of the electric field and the ϕ component of the magnetic field are known throughout the whole volume $r < R$ and are given by

$$E_{ynl}^- = \frac{J_l(\Gamma_n r)}{J_l(\Gamma_n R)} e^{jl\phi} \cos(k_{yn} y)$$

$$H_{\phi nl}^- = \frac{-jk_3}{Z_3 \Gamma_n} \frac{J_l'(\Gamma_n r)}{J_l(\Gamma_n R)} e^{jl\phi} \cos(k_{yn} y). \tag{A3}$$

Using (A1)–(A3) it can be shown that the scattering coefficients s_{nlm} and the current expansion coefficients are related by

$$s_{nlm} = \frac{-jk_3}{Z_3 \Gamma_n} \frac{J_l'(\Gamma_n R)}{J_l(\Gamma_n R)} \delta_{lm} + J_{nlm}. \tag{A4}$$

This last relationship indicates that the determination of the scattering coefficients can be reduced to the determination of the expansion coefficients of the current at $r = R$ which produces the y component of the electric field at $r = c$, given by (A3). The equation for unknown expansion coefficients can be rewritten in the following form:

$$\frac{1}{2\pi} \int_0^{2\pi} E_{yn}(J_{nl}) e^{-jp\phi} \, d\phi = \delta_{lp} \frac{J_l(\Gamma_n c)}{J_l(\Gamma_n R)} \cos(k_{yn} y), \quad p = -L, \cdots, L \tag{A5}$$

where $E_{yn}(J_{nl})$ is the y component of electric field at $r = c$ due to a given distribution of the current J_{nl} at $r = R$.

For J_{nl} of the form (A2) expression (A5) generates a system of linear algebraic equations for the expansion coefficients J_{nlm}. To avoid any problems with field singularities the radius c in (A5) can be taken in the range $0.2R$–$0.8R$ [8]. The expression of the y component of the electric field due to a given current can be obtained by using a modal expansion of the field. For the rectangular waveguide being matched at one arm and loaded at the other, the y component of the electric field E_{yn} at (x_1, z_1) due to a filament of current $I_n(y) = \cos(k_{yn} y)$ located at (x_2, z_2) is given by

$$\begin{aligned} E_{yn}(x_1, z_1) = \frac{-jZ_3\Gamma_n^2}{Ak_3} \Bigg\{ & \sum_{m=1}^{\infty} \Bigg[\frac{e^{-\Gamma_{mn}|z_2 - z_1|}}{\Gamma_{mn}} - \frac{e^{-k_{xm}|z_2 - z_1|}}{k_{xm}} + \mathrm{RC} \frac{e^{-\Gamma_{mn}(2u + (z_1 + z_2))}}{\Gamma_{mn}} \Bigg] \\ & \cdot \sin(k_{xm} x_1) \sin(k_{xm} x_2) \\ & + \frac{A}{2\pi} \mathrm{Re} \left[\ln \frac{1 - e^{j(\pi/A)((x_1 + x_2) + j|z_2 - z_1|)}}{1 - e^{j(\pi/A)((x_1 - x_2) + j|z_2 - z_1|)}} \right] \Bigg\} \\ & \cdot \cos(k_{yn} y) \end{aligned} \tag{A6}$$

where $k_{xm} = m\pi/A$ and $\Gamma_{mn}^2 = k_{xm}^2 + k_{yn}^2 - k_3^2$. RC is a reflection coefficient of the load located at a distance u from the middle position of the mount. Note that the convergence of the series in (A6) has been accelerated by adding and subtracting slowly converging residual series whose sum can be expressed in terms of elementary functions.

The required values of the integrals in (A5) can be evaluated by using (A6) and a stepwise approximation. Having solved the system (A5), the scattering coefficients can be found using (A4). For n sufficiently large (usually for $n > 2$) such that $\Gamma_n^2 < 0$ a simplified approach for determining the scattering coefficients can be used. For $\Gamma_n^2 < 0$ radial waves generated by currents having dependence $\cos(k_{yn} y)$ on y decay with distance and therefore their interaction with waveguide walls can be neglected. In such a case the rectangular waveguide can be substituted by a parallel-plate guide and the scattering coefficients can be approximated by

$$s_{nlm} = \delta_{lm} \frac{jk_3}{Z_3 q_n} \frac{K_l'(q_n R)}{K_l(q_n R)} \tag{A7}$$

where $q_n^2 = -\Gamma_n^2$ and K_l is a modified Bessel function.

B. Scattering of TE Waves

The problem of determining the scattering coefficients for the TE radial waves can be regarded as the dual of that for solving for the TM waves. The TE radial waves can be considered as due to a magnetic current located on the cylinder $r = R$ and flowing in the y direction. The scattering coefficient f_{nlm} is defined as the magnitude of the nmth harmonic of the y component of the magnetic field while the external ϕ component of the electric field at $r = R$ is given by the nlth harmonic $E^+_{\phi nl}$ of unit magnitude. The magnetic current density producing $E^+_\phi = E^+_{\phi nl}$ can be expanded into the series

$$M_{nl} = \sum_{m=-L}^{L} M_{nlm} e^{jm\phi} \sin(k_{yn} y) \tag{A8}$$

where M_{nlm} are expansion coefficients.

For the assumed value of the external field at $r = R$ the internal y component of the magnetic field and the ϕ component of the electric field are given by

$$H^-_{ynl} = \sum_{l=-L}^{L} f_{nlm} \frac{J_m(\Gamma_n r)}{J_m(\Gamma_n R)} e^{jm\phi} \sin(k_{yn} y)$$

$$E^-_{\phi nl} = \frac{jk_3 Z_3}{\Gamma_n} \sum_{m=-L}^{L} f_{nlm} \frac{J'_m(\Gamma_n r)}{J_m(\Gamma_n R)} e^{jm\phi} \sin(k_{yn} y). \tag{A9}$$

By using (22), (23), and continuity conditions it can be shown that the scattering coefficient and the current expansion coefficient are related by

$$f_{nlm} = \delta_{lm} - \frac{jk_3 Z_3}{\Gamma_n} \frac{J'_m(\Gamma_n R)}{J_m(\Gamma_n R)} M_{nlm}. \tag{A10}$$

The last expression indicates that the determination of the scattering coefficients can be reduced to the determination of the expansion coefficients for the current. The latter can be found by solving the problem for the magnetic current located at $r = R$ which produces the y component of the magnetic field at $r = c$ given by (A9).

The equation for the unknown current can be written in the form

$$\frac{1}{2\pi} \int_0^{2\pi} H_{yn}(M_{nl}) e^{-jp\phi} d\phi = M_{nlp} \frac{J_p(\Gamma_n c)}{J_p(\Gamma_n R)} \sin(k_{yn} y), \quad p = -L, \cdots, L \tag{A11}$$

where $H_{yn}(M_{nl})$ is the y component of the magnetic field at $r = c$ due to the current M_{nl} at $r = R$. For M_{nl} of (A8) expression (A11) generates a linear system of algebraic equations for the coefficients M_{nlm}.

To evaluate the integrals in (A11) a stepwise approximation and a modal expansion of the y component of the magnetic field can be used. In this case the required y component of the magnetic field at (x_1, z_1) due to a filament of the magnetic current $M_n(y) = \sin(k_{yn} y)$ at (x_2, z_2) is given by

$$\begin{aligned} H_{yn}(x_1, z_1) = \frac{j\Gamma_n^2}{AZ_3 k_3} \Bigg\{ \sum_{m=1}^{\infty} \Bigg[& \frac{e^{-\Gamma_{mn}|z_2 - z_1|}}{\Gamma_{mn}} - \frac{e^{-k_{xm}|z_2 - z_1|}}{k_{xm}} - \mathrm{RC} \frac{e^{-\Gamma_{mn}(2u + (z_1 + z_2))}}{\Gamma_{mn}} \Bigg] \\ & \cdot \cos(k_{xm} x_1) \cos(k_{xm} x_2) \\ & + \frac{1}{2} \left(\frac{e^{-\Gamma_{0n}|z_2 - z_1|}}{\Gamma_{0n}} - \mathrm{RC} \frac{e^{-\Gamma_{0n}(2u + (z_1 + z_2))}}{\Gamma_{0n}} \right) \\ & - \frac{A}{2\pi} \mathrm{Re} \Big[\ln \Big(\big(1 - e^{j(\pi/A)((x_1 + x_2) + j|z_2 - z_1|)}\big) \\ & \cdot \big(1 - e^{j(\pi/A)((x_1 - x_2) + j|z_2 - z_1|)}\big) \Big) \Big] \Bigg\} \sin(k_{yn} y). \end{aligned} \tag{A12}$$

The convergence of the series in (A12) has been accelerated by adding and subtracting the slowly converging residual series.

Having determined the expansion coefficients for the current the scattering coefficients are found using (A10). Similarly, as for the TM waves, the scattering coefficients of the higher order TE waves can be found by using a parallel-plate approximation. For waves such that $\Gamma_n^2 < 0$ the scattering coefficients f_{nlm} are given by

$$f_{nlm} = \delta_{lm} \frac{jq_n}{k_3 Z_3} \frac{K_l(q_n R)}{K'_l(q_n R)}. \tag{A13}$$

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Marek E. Bialkowski (SM'89) was born in Sochaczew, Poland, in 1951. He received the M.Eng.Sc. degree (1974) in applied mathematics and the Ph.D. degree (1979) in electrical engineering, both from the Warsaw Technical University, Warsaw, Poland.

In 1977 he joined the Institute of Radioelectronics, Warsaw Technical University, as a Teaching Assistant and in 1979 became an Assistant Professor there. In 1981, he was awarded a Postdoctoral Research Fellowship by the Irish Department of Education and spent one year at the University College Dublin carrying out research in the area of microwave circuits. In 1982, he won a Postdoctoral Research Fellowship from the University of Queensland, Brisbane, Australia. During his stay in Brisbane he worked on the modeling of millimeter-wave guiding structures and microwave measurements. In 1984, he joined the Department of Electrical and Electronic Engineering, James Cook University of North Queensland, Townsville, Australia, as a Lecturer in the field of communications. In 1986 he was promoted to Senior Lecturer. In 1988 he was a Visiting Lecturer in the Department of Electronics and Computer Science, University of Southampton, U.K. He was invited to lecture in the field of antenna theory and design. Recently, he accepted an appointment as Reader (Associate Professor) in the Department of Electrical Engineering at the University of Queensland, Brisbane, Australia.

Dr. Bialkowski's current interests include six-port techniques, electromagnetic models of waveguide diode mounts, near field–far field antenna measurements, the design of monomode and multimode feeds for dish antennas, and the design of computer-controlled microwave located devices for industrial applications.